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# Analysis of the Backscatter Spectrum in an Ionospheric Modification Experiment

by  
Hongjin Kim

June 1973

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ABSTRACT

The purpose of this study is to compare predictions of the backscatter spectrum, including effects of ionospheric inhomogeneity, with experimental observations of incoherent backscatter from an artificially heated region. Our calculations show that the strongest backscatter echo received is not, in fact, from the reflection level, but from a region some distance below (about 0.5 km for an experiment carried out at Arecibo), where the pump wave from a HF transmitter ( $\sim 100$  kW) is below the threshold for parametric amplification. By taking the standing wave pattern of the pump into account properly, the present theory explains the asymmetry of the up-shifted and down-shifted plasma lines in the backscatter spectrum, and the several peaks typically observed in the region of the spectrum near the HF transmitter frequency.

## I. INTRODUCTION

From experimental results obtained at Arecibo (Carlson, Gordon and Showen, 1972; Kantor, 1972), it is believed that the enhanced heating and anomalous absorption occurring in the ionosphere are mainly due to parametric decay instabilities; the HF ordinary (pump) wave propagating in the F-layer from a high power transmitter ( $\sim 100$  kW) decays into electron plasma and ion acoustic waves to enhance the plasma fluctuations (DuBois and Goldman, 1965; Nishikawa, 1968). Several analyses have been carried out of the saturation spectra in homogeneous (DuBois and Goldman, 1972a and b; Valeo, Oberman and Perkins, 1972; Kruer and Valeo, 1972; Harker, 1972; Kuo and Fejer, 1972; Fejer and Kuo, 1972; Perkins and Valeo, 1973) or inhomogeneous (Arnush, Fried and Kennel, 1972a and b) plasmas, with the assumption that the pump field is well above the threshold for parametric amplification. This results in saturation of the electron plasma waves due to their nonlinear Landau damping by positive ions, or due to their absorption by electrons whose orbits are nonlinearly perturbed by the waves (Bezzerides and Weinstock, 1972).

However, some essential features of the backscatter spectrum still remain to be explained. For example, Carlson et al. (1972) have obtained a pair of plasma lines in the spectrum enhanced by the pump wave ( $f_0 = 5.62$  MHz). These plasma lines are shifted up and down from the incoherent scatter diagnostic beam ( $f_i = 430$  MHz) by  $f_0 - f_a$ , where  $f_a$  ( $\approx 4$  kHz) is the frequency of the ion acoustic waves generated, and differ from each other in the intensities of the peaks. As shown in Fig. 1, Kantor (1972) has also observed several maxima and minima near the peaks of the spectrum. So far, these features of the spectrum have not been explained.

The purpose of the present analysis is to take the inhomogeneity of the ionosphere into account and to compare our predictions with observations of incoherent scatter. In contrast to most of the previous analyses, our theory assumes that the pump is below the threshold. Section II is concerned with the pump wave. Its intensity is derived as a function of height. In Section III, the intensity of the parametrically enhanced

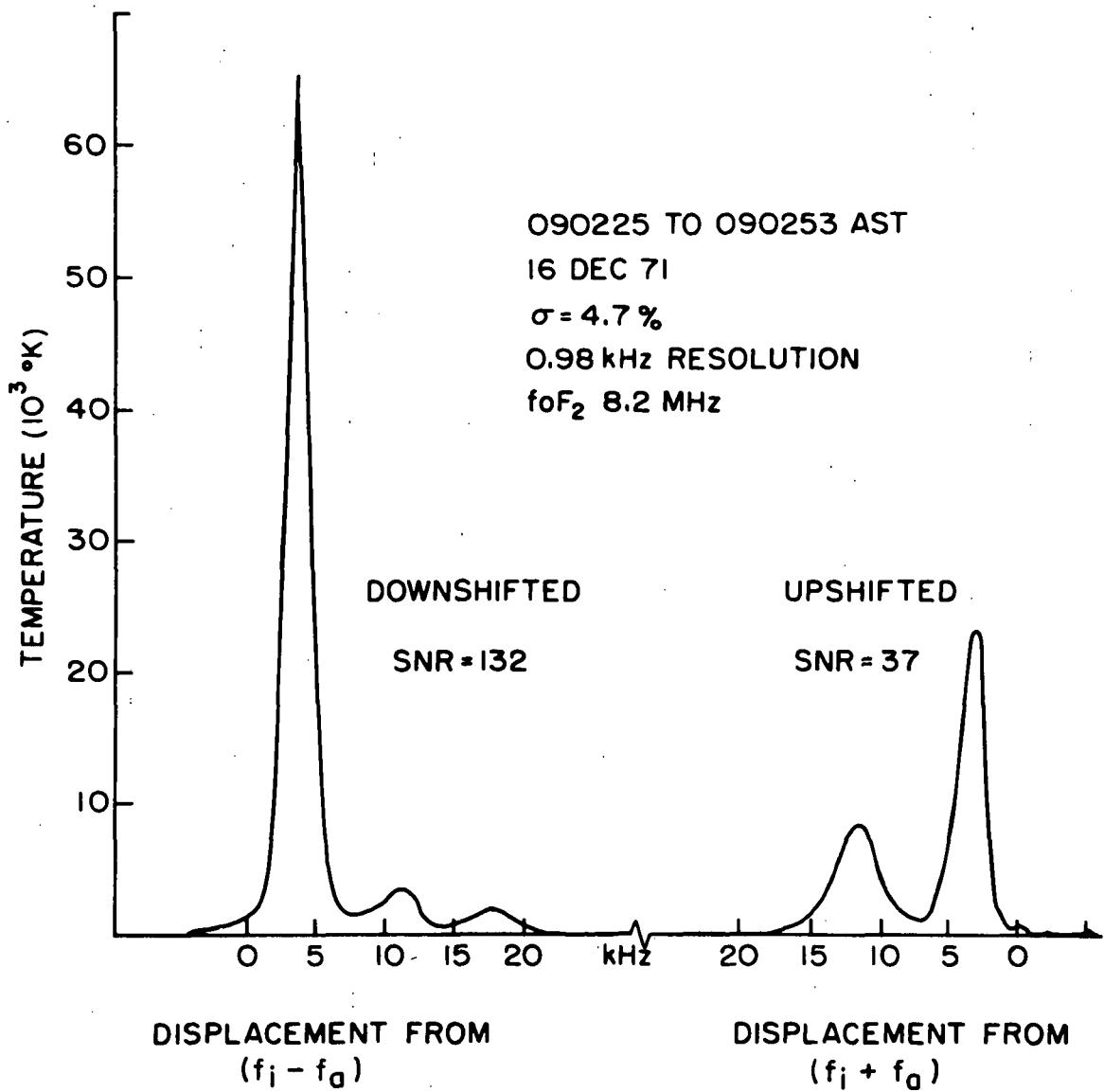


FIG. 1. Frequency spectrum obtained by incoherent backscatter at Arecibo (Kantor, 1972).

electron plasma wave is obtained from the kinetic wave equation. Section IV is devoted to the incoherent scatter experiment performed at Arecibo, and frequency spectra are obtained for several cases. Our assumptions are discussed and predictions are compared to experimental data in Section V.

## II. INTENSITY OF THE PUMP WAVE

As the pump wave from the HF transmitter propagates vertically upward in the ionosphere, its group velocity decreases very sharply near, and reaches zero at, the reflection level, as shown in Fig. 2. Since more slowly moving waves spend a longer time in a location, and are thus able to exchange energies more effectively, we shall assume that energy from the pump wave is deposited in the ionosphere only at or very near its reflection level and we neglect the depletion of its intensity due to the linear or nonlinear loss in the region of interest. The present investigation is primarily concerned with heating at frequencies below the penetration frequency, so that the electron density profile can be approximated by a linear profile, as shown in Fig. 2(a). For simplicity, we neglect the geomagnetic field. After the pump field is found, however, it is regarded as being aligned in the direction of the geomagnetic field.

A vertically propagating ordinary wave, of angular frequency  $\omega_0$ , is described by the wave equations

$$\frac{d^2 E}{dz^2} + \left( \frac{\omega_0 h_0}{c} \right)^2 \epsilon(z) E = 0, \quad \text{for } z \geq 0 \text{ (in the ionosphere)}$$

$$\frac{d^2 E}{dz^2} + \left( \frac{\omega_0 h_0}{c} \right)^2 E = 0, \quad \text{for } z < 0 \text{ (in the free space)}, \quad (1)$$

where  $z$  is the height measured from the base of the ionosphere ( $z = 0$ ), and is normalized to the scale height  $h_0$ ;  $c$  is the speed of light; the wave magnetic field is related to the electric field by

$$B = \frac{1}{i\omega_0 h_0} \frac{dE}{dz}, \quad (2)$$

and the plasma equivalent permittivity,  $\epsilon(z)$ , is given by

$$\epsilon(z) = 1 - \frac{p}{\omega_0^2} = 1 - z. \quad (3)$$

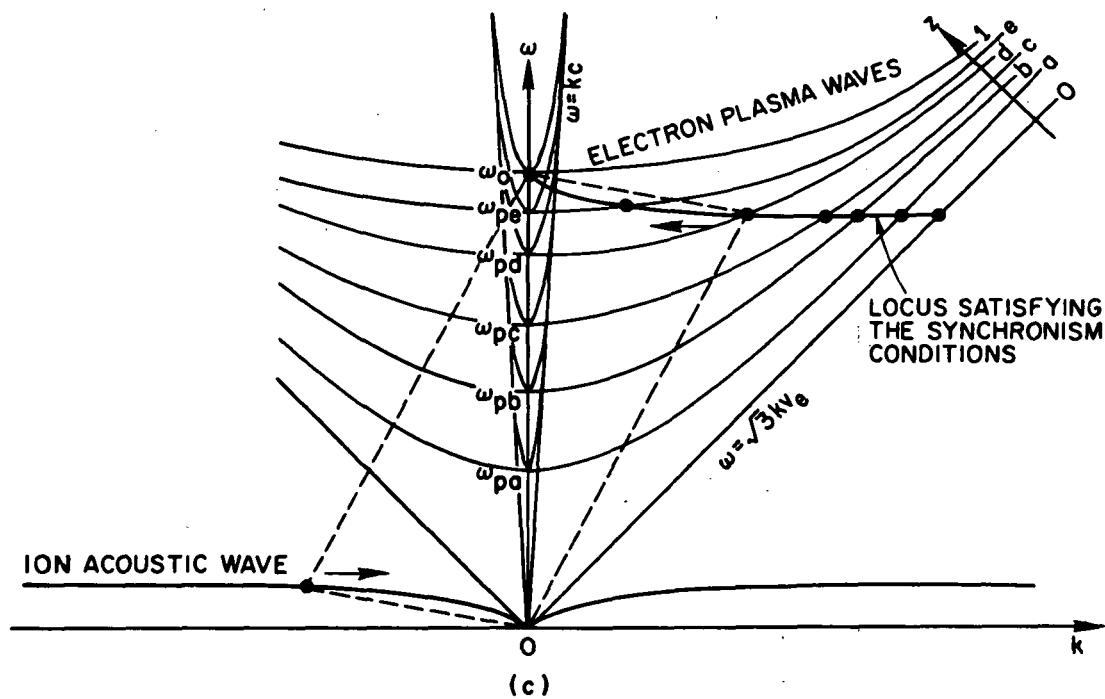
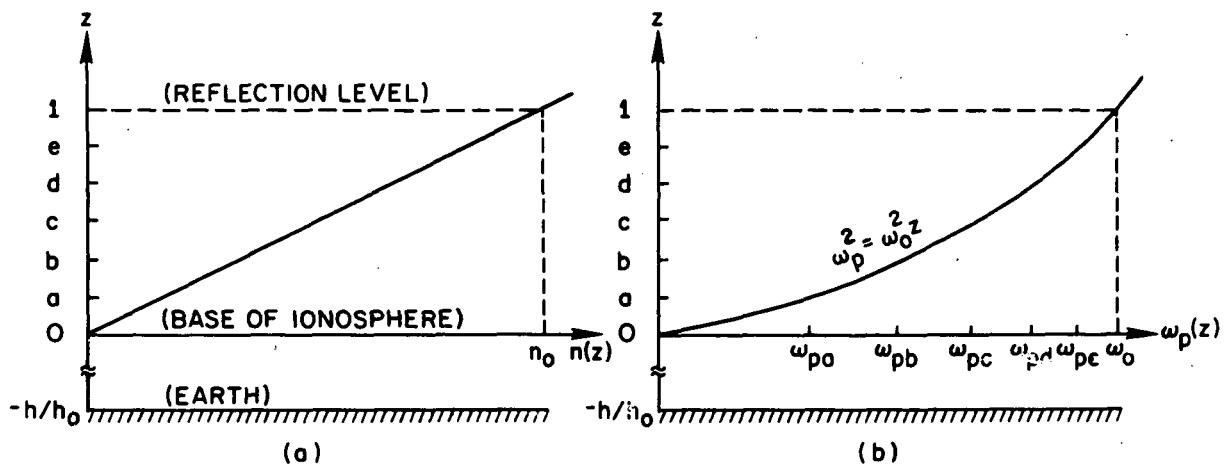


FIG. 2. The electron density and the electron plasma frequency are shown in (a) and (b), respectively, as functions of  $z$ . (c) shows the synchronism parallogram for decay instability at different heights in the ionosphere.

The solution to Eq. (1) yields

$$E = a \text{Ai}(-\zeta) + b \text{Bi}(-\zeta), \quad \text{for } z \geq 0, \quad (4)$$

$$E = E_0 \left[ \exp(-i\zeta_0^{3/2}z) + R \exp(i\zeta_0^{3/2}z) \right], \quad \text{for } z < 0,$$

where  $\text{Ai}$  and  $\text{Bi}$  are the Airy integral functions (Abramowitz and Stegun, 1964);  $R$  is the reflection coefficient, and the variables  $\zeta$  and  $\zeta_0$  are defined by

$$\zeta = \zeta_0(1-z), \quad \zeta_0 = \left( \frac{\omega_0 h_0}{c} \right)^{2/3}. \quad (5)$$

The magnitude of the electric field intensity of the incident pump wave at  $z = 0$ ,  $E_0$ , is given by

$$E_0 = \left( 60 \frac{P_0 G}{h^2} \right)^{1/2} \quad (\text{V/m}), \quad (6)$$

where the average heater power  $P_0$  is in watts;  $h$  is the height of the base of the ionosphere from the ground, and the gain,  $G$ , of the antenna is given by

$$G = \frac{2\pi A}{c^2} f_0^2, \quad (7)$$

where  $A$  is the aperture. In the absence of anomalous absorption, it is customary to put  $b = 0$  in (4) because the electric field has to vanish as  $z \rightarrow \infty$  (Ginzberg, 1964; Budden, 1961). In the present situation, where most of the pump wave energy is deposited at or near the reflection level, however, we assume that the electric field vanishes in  $z \geq 1$  due to absorption, and phenomenologically keep the  $\text{Bi}$  term in the region below the reflection level ( $0 \leq z < 1$ ).

Application of the boundary conditions of continuity of  $E$  and  $B$  at  $z = 0$ , and use of (2), reduce (4) to the form

$$|E|^2 = S |E_0|^2 , \quad (8)$$

where the swelling factor,  $S$ , is given by

$$S = \pi \zeta_0^{1/2} \left\{ \exp \left[ -i \left( \frac{2}{3} \zeta_0^{3/2} - \frac{\pi}{4} \right) \right] + R \exp \left[ i \left( \frac{2}{3} \zeta_0^{3/2} - \frac{\pi}{4} \right) \right] \right\} \text{Ai}(-\zeta) \\ + \left\{ \exp \left[ -i \left( \frac{2}{3} \zeta_0^{3/2} + \frac{\pi}{4} \right) \right] + R \exp \left[ i \left( \frac{2}{3} \zeta_0^{3/2} + \frac{\pi}{4} \right) \right] \right\} \text{Bi}(-\zeta) \Big|^2 , \quad (9)$$

and use has been made of (Abramowitz and Stegun, 1964)

$$\text{Ai}(x)\text{Bi}'(x) - \text{Ai}'(x)\text{Bi}(x) = \frac{1}{\pi} . \quad (10)$$

Noting that  $\zeta_0 \gg 1$ , use has also been made of the following asymptotic expressions for the Airy integral functions valid for  $|x| \gg 1$

$$\text{Ai}(-x) \approx \frac{1}{\pi^{1/2}} x^{-1/4} \sin \left( \frac{2}{3} x^{3/2} + \frac{\pi}{4} \right) , \quad \text{Ai}'(-x) \approx - \frac{1}{\pi^{1/2}} x^{1/4} \cos \left( \frac{2}{3} x^{3/2} + \frac{\pi}{4} \right) ,$$

$$\text{Bi}(-x) \approx \frac{1}{\pi^{1/2}} x^{-1/4} \cos \left( \frac{2}{3} x^{3/2} + \frac{\pi}{4} \right) , \quad \text{Bi}'(-x) \approx \frac{1}{\pi^{1/2}} x^{1/4} \sin \left( \frac{2}{3} x^{3/2} + \frac{\pi}{4} \right) . \quad (11)$$

The familiar results of  $S$  and  $R$  in the absence of the absorption are recovered from (9) by letting the  $\text{Bi}$  term equal zero (Ginzberg, 1964; Budden, 1961), as

$$S = 4\pi \zeta_0^{1/2} \text{Ai}^2(-\zeta) , \quad R = \exp \left[ -i \left( \frac{4}{3} \zeta_0^{3/2} - \frac{\pi}{2} \right) \right] . \quad (12)$$

At the reflection level ( $\zeta = 0$ ), the swelling factor becomes

$$S = 4\pi c_0^2 \zeta_0^{1/2} \left[ 1 + |R|^2 + 2|R| \cos \left( \frac{4}{3} \zeta_0^{3/2} + \frac{\pi}{6} + \psi \right) \right] , \\ c_0 \equiv \text{Ai}(0) = 3^{-1/2} \text{Bi}(0) = \frac{3^{-2/3}}{(-1/3)!} , \quad (13)$$

where  $\psi$  is the phase of the reflection coefficient,  $R$ . The condition that the WKB solution be a good approximation to (1) is  $|\zeta| \gg 1$ . In this region, (9) becomes

$$S = \left(\frac{\zeta_0}{\zeta}\right)^{1/2} \left\{ 1 + |R|^2 + 2|R| \cos \left[ \frac{4}{3} \left( \zeta_0^{3/2} - \zeta^{3/2} \right) + \psi \right] \right\}. \quad (14)$$

Figure 2 shows how the swelling factor varies with height for small  $|R|$ , in the region where the WKB approximation is valid.

The reflection coefficient,  $R$ , can be measured from the echo of the pump wave. Hence, the field intensities of the pump wave at the reflection level, and in the region where the WKB approximation is valid, are determined by (8) with the swelling factors given by (13) and (14), respectively.

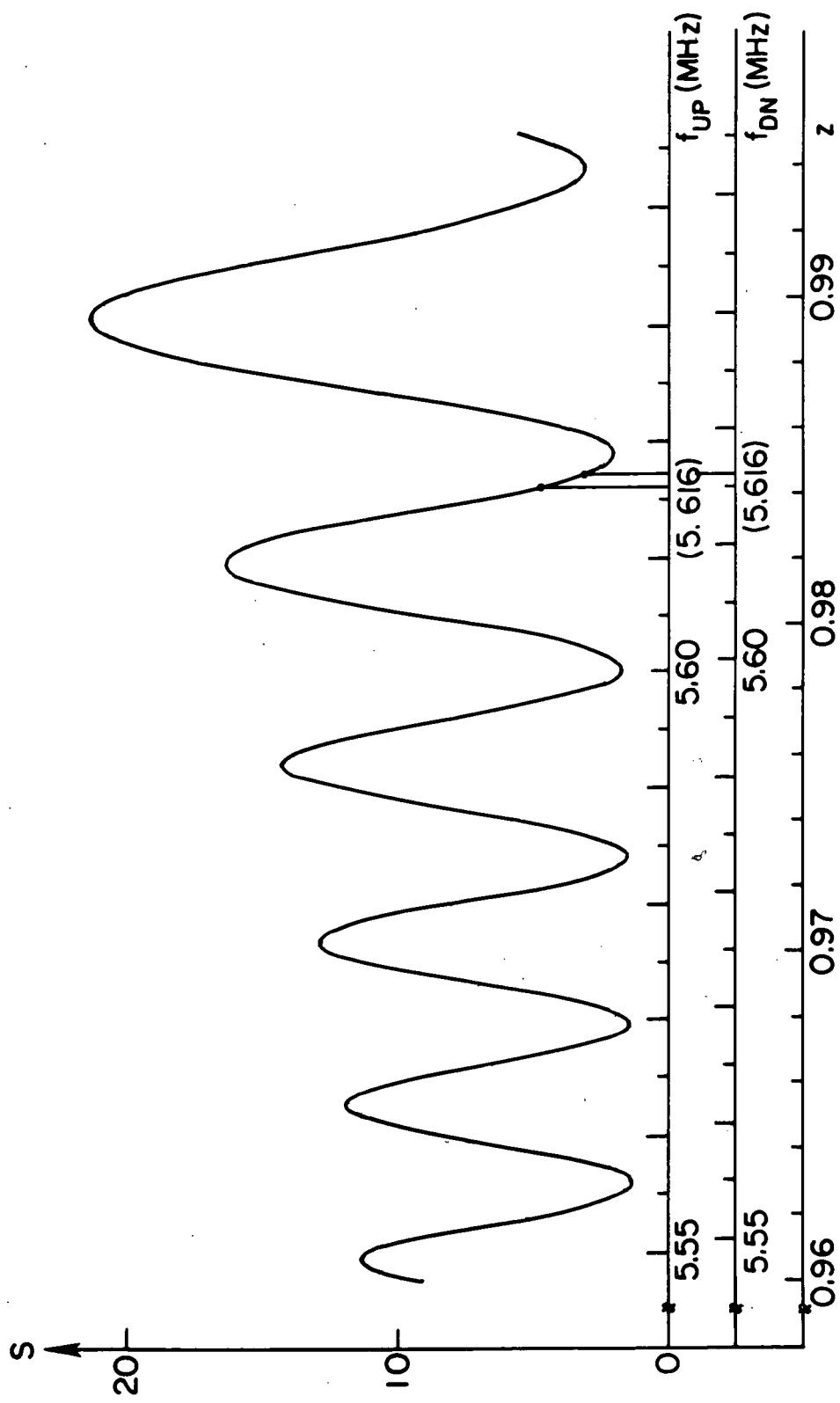


FIG. 3. Swelling factor for  $|R| = 1/2$  and  $\psi = 0$  as function of  $z$ . The frequency scales are obtained from (25).

### III. KINETIC WAVE EQUATION FOR THE ENHANCED PLASMA WAVE

Figure 2(c) shows a set of electron plasma wave dispersion waves at different heights ( $z = 0$  to  $1$ ) and a locus satisfying the synchronism conditions for parametric interaction among the pump wave ( $\omega_0, \underline{k}_0$ ), the electron plasma wave ( $\omega, \underline{k}$ ) and the ion acoustic wave ( $\omega_a, \underline{k}_a$ ). As shown in the figure, the electron plasma wave may be excited at any height in the ionosphere. We assume that the pump field is below threshold so that higher order processes such as the nonlinear ion Landau damping or electron orbit perturbations are negligible. The kinetic wave equation describing the excited electron plasma wave may then be written in a WKB approximation form as (DuBois and Goldman, 1972a; Valeo, Oberman and Perkins, 1972)

$$\frac{1}{2\omega} \left( \frac{\partial}{\partial t} + \underline{v}_g \cdot \frac{\partial}{\partial \underline{r}} \right) \underline{\mathcal{E}}_k = \left[ \left( \frac{\omega_0}{\omega_p} \right)^2 \frac{\underline{\mathcal{E}}_0 \underline{s}_\mu^2}{1+d^2} - \frac{\gamma_e}{\omega} \right] \underline{\mathcal{E}}_k + \frac{\alpha k \gamma_e}{k_D \omega} + \left( \frac{\omega_0}{\omega_p} \right)^2 \frac{\underline{\mathcal{E}}_0 \underline{s}_\mu^2}{1+d^2}, \quad (15)$$

where  $\underline{v}_g$  is the group velocity of the electron plasma waves;  $\alpha$  is the square root of the ion electron mass ratio;  $k_D$  is the Debye wave number;  $\mu$  is the direction cosine of  $\underline{k}$  with respect to the pump field  $\underline{E}_0$  which is aligned with the geomagnetic field;  $\gamma_e$  and  $\gamma_a$  are the linear damping rates of the electron plasma and ion acoustic waves, respectively, and

$$\underline{\mathcal{E}}_k = \left( \frac{\alpha k}{4\pi k_D} \right)^{\frac{1}{2}} \underline{\mathcal{I}}_k, \quad \underline{\mathcal{E}}_0 = \frac{\underline{\mathcal{E}}_0}{16n_0^{\frac{1}{2}}} |\underline{E}_0|^2, \quad d = \frac{\omega_0 - \omega - \omega_a}{\gamma_a}. \quad (16)$$

We define  $n_0$  as the electron density at the reflection height;  $\Theta$  as the electron temperature in energy units, and

$$\underline{\mathcal{I}}_k = \lim_{V, T \rightarrow \infty} \frac{\int d\omega |\underline{E}_k(\underline{k}, \omega)|^2}{2\pi V T} \quad (17)$$

as the spectral density of the electron plasma waves. The dispersion relations for the plasma and ion acoustic waves are given approximately by

$$\omega^2 = \omega_p^2 \left[ 1 + \beta \left( \frac{k}{k_D} \right)^2 \right], \quad \omega_a = \alpha k v_e \approx \gamma_a, \quad (18)$$

where  $v_e$  is the electron thermal velocity.

The first and second (bracketed) terms on the RHS of (15) describe the parametric growth due to the pump wave and the linear damping, respectively. The third term describes the thermal fluctuations, which are small compared to the Cerenkov radiation term, the last one. Hence, we neglect the third term (Valeo, Oberman and Perkins, 1972).

We now assume that the electron plasma wave has reached a constant amplitude at which the parametric growth is balanced by linear damping and energy loss due to propagation out of the interaction region. By using the expression for the group velocity

$$v_g = \beta^{1/2} v_e \left[ 1 - \left( \frac{\omega_0}{\omega} \right)^2 z \right]^{1/2}, \quad (19)$$

Eq. (15) may be rewritten as

$$Q \left[ \left( \frac{\omega}{\omega_0} \right)^2 - z \right]^{1/2} \frac{d\epsilon_k}{dz} = (PS-1) \epsilon_k + PS, \quad (20)$$

$$P = \frac{\omega}{\gamma_e} \left( \frac{\omega_0}{\omega_p} \right)^2 \frac{\epsilon_0 \mu^2}{1+d^2}, \quad Q = \frac{\beta^{1/2} \mu' v_e \omega_0}{2 \gamma_e H \omega},$$

where the direction cosine of  $\mathbf{k}$  with respect to the  $z$ -axis,  $\mu'$ , is related to  $\mu$  by

$$\mu' = \mu \cos \theta_0 - \sin \theta \sin \theta_0 \cos \varphi, \quad (21)$$

where  $\theta_0$  is the angle between the geomagnetic field and the vertical direction.

Since  $P \sim (1+d^2)^{-1}$  in (20), the generated electron plasma wave has its peak near  $\omega = \omega_0 - \omega_a$  at any height. For a given  $\omega$ , numerical solution for  $\mathcal{E}_k(z)$  is straightforward. The required boundary condition is  $\mathcal{E}_k(0) = 0$  because no electron plasma wave can be generated at the base of the ionosphere.

#### IV. THE INCOHERENT SCATTER SPECTRUM

An incident diagnostic wave propagating vertically upward in the ionosphere is scattered by the enhanced electron plasma waves of (20). The scattered wave detected by the receiver should satisfy the synchronism conditions (Beketi, 1966)

$$\omega_s = \omega_i \pm \omega, \quad k_s = k - k_i \quad \left\{ \begin{array}{l} \text{up-shifted} \\ \text{down-shifted} \end{array} \right\} \quad . \quad (22)$$

For incident and scattered waves with frequencies very high compared to the penetration frequency of the ionosphere,  $\omega_{i,s}$  is related to  $k_{i,s}$  by the simple dispersion relation

$$\omega_{i,s} = k_{i,s} c \quad . \quad (23)$$

The loci in the  $\omega, k$ -plane satisfying the synchronism conditions are obtained by eliminating  $k_{i,s}$  and  $\omega_s$  from (22) and (23) as

$$k = \frac{2\omega_i}{c} \pm \frac{\omega}{c} \quad \left\{ \begin{array}{l} \text{up-shifted} \\ \text{down-shifted} \end{array} \right\} \quad . \quad (24)$$

These loci are shown by heavy lines in Figure 4. Since  $\omega \ll 2\omega_i$ , the last term on the RHS of (24) is usually dropped and the loci are considered as vertical lines at  $k = \pm 2k_i$ . However, it turns out to be an important term which causes the difference in magnitudes and shapes of up-shifted and down-shifted spectra.

Figure 4 also shows two dispersion curves of electron plasma waves with angular frequencies  $\omega = \omega_0 - \omega_a$  for  $k$  given by (24). Since, as shown in (20), the electron plasma waves are enhanced most significantly near  $\omega = \omega_0 - \omega_a$ , these two are among the most important waves. The figure clearly shows that they belong to different heights. The dependence of the frequency of electron plasma waves on height is found from (18) and (24) as

$$z(\omega) = \left( \frac{\omega}{\omega_0} \right)^2 - \frac{12 \omega_i (\omega_i \pm \omega_0)}{\omega_0^2} \left( \frac{v_e}{c} \right)^2 \quad \left\{ \begin{array}{l} \text{up-shifted} \\ \text{down-shifted} \end{array} \right\} \quad , \quad (25)$$

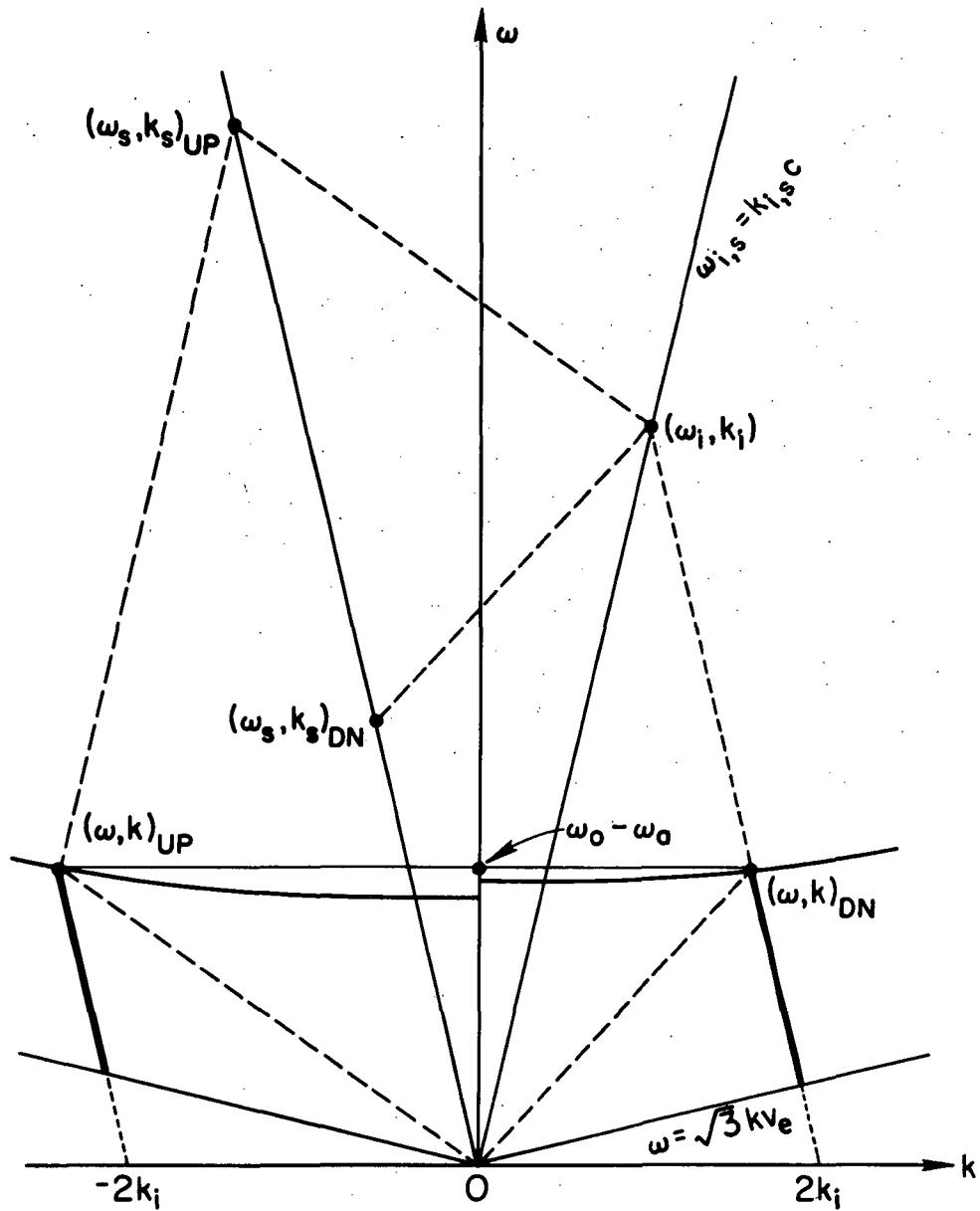


FIG. 4. The synchronism parallelogram for up- and down-shifted plasma lines.

where, since  $\omega_i \gg \omega_0 \gg \omega_a$ , the value of  $k$  in (24) has been taken at  $\omega = \omega_0$ . The heights far above two electron plasma waves are then found from (25) with  $\omega = \omega_0 - \omega_a$ : For typical values in the Arecibo experiment of  $f_0 = 5.62$  MHz,  $f_i = 430$  MHz,  $f_a = 4$  kHz,  $T_e = 1200^{\circ}$ K and  $H = 3 \times 10^4$  m, they are

$$h_M \approx H - \left\{ \begin{array}{l} 4.75 \\ 4.64 \end{array} \right\} \times 10^2 \text{ m} , \quad \left\{ \begin{array}{l} \text{up-shifted} \\ \text{down-shifted} \end{array} \right\} . \quad (26)$$

The height where the maximum up-shifted plasma line comes from is about 10 m below that for the maximum down-shifted plasma line. The magnitudes of the swelling factor at these heights are shown in Figure 3. Consequently, different intensities of the electron plasma waves at these heights are expected from (20), which in turn result in different magnitudes of up-shifted and down-shifted spectra. The power scattered by the enhanced plasma waves is given by (Harker, 1972) as

$$\frac{dP_s}{dz} = \frac{n\sigma_T}{\alpha h_0} \left( \frac{k}{k_D} \right) \frac{P_t A_r}{(z+h/h_0)^2} \epsilon_k dz , \quad (27)$$

where  $A_r$  is the receiver aperture area;  $P_t$  is the transmitted radar power, and the Thompson backscattering cross-section,  $\sigma_T$ , may be taken as the square of the classical electron radius for the present purpose,

$$\sigma_T = r_0^2 = 7.94 \times 10^{-30} \text{ m}^2 . \quad (28)$$

From (25), the variable  $z$  may be changed to  $\omega$  via

$$dz = 2 \frac{\omega}{\omega_0^2} d\omega . \quad (29)$$

Equation (27) then yields

$$\frac{dP_s}{d\omega} = \frac{2n_0 \sigma_T v_e P_t A_r}{\alpha c h_0 \omega_0^3} \frac{(2\omega_i \pm \omega)\omega}{\left[ \frac{z^{1/2}}{(z+h/h_0)^2} \right]} \epsilon_k [z(\omega)] , \quad (30)$$

where  $n_0$  is  $n$  at the reflection height. The power spectrum is plotted in Figure 5, with the phase of the reflection coefficient as a parameter, and with typical values for the Arecibo experiment:

$$\begin{aligned} T &= 1200^{\circ}\text{K} , \quad H = 30 \text{ km} , \quad h = 150 \text{ km} , \\ f_0 &= 5.62 \text{ MHz} , \quad \theta_0 = 40^{\circ} , \\ f_a &= \gamma_a / 2\pi = 4 \text{ kHz} , \quad P_0 = 100 \text{ kW} , \\ f_e &= \gamma_e / 2\pi = 650 \text{ Hz} , \quad P_T = 2.5 \text{ MW} , \\ f_i &= 430 \text{ MHz} , \quad R = 1/2 , \\ \text{Antenna diameter} &= 300 \text{ m} , \quad \text{Antenna efficiency} = 40\% . \end{aligned} \tag{31}$$

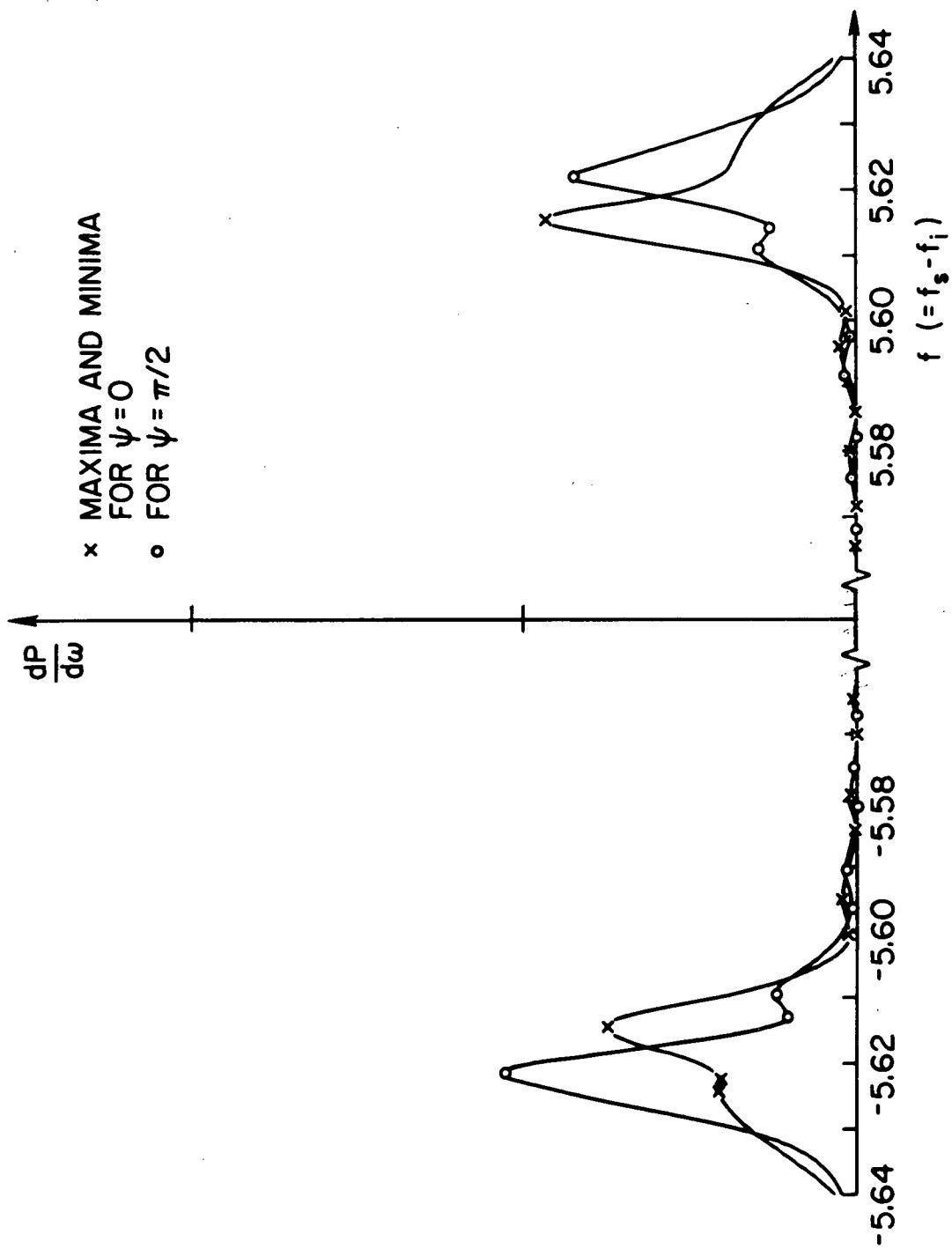


FIG. 5. Frequency spectra (a) when  $\psi = 0$  and  $\pi/2$ .

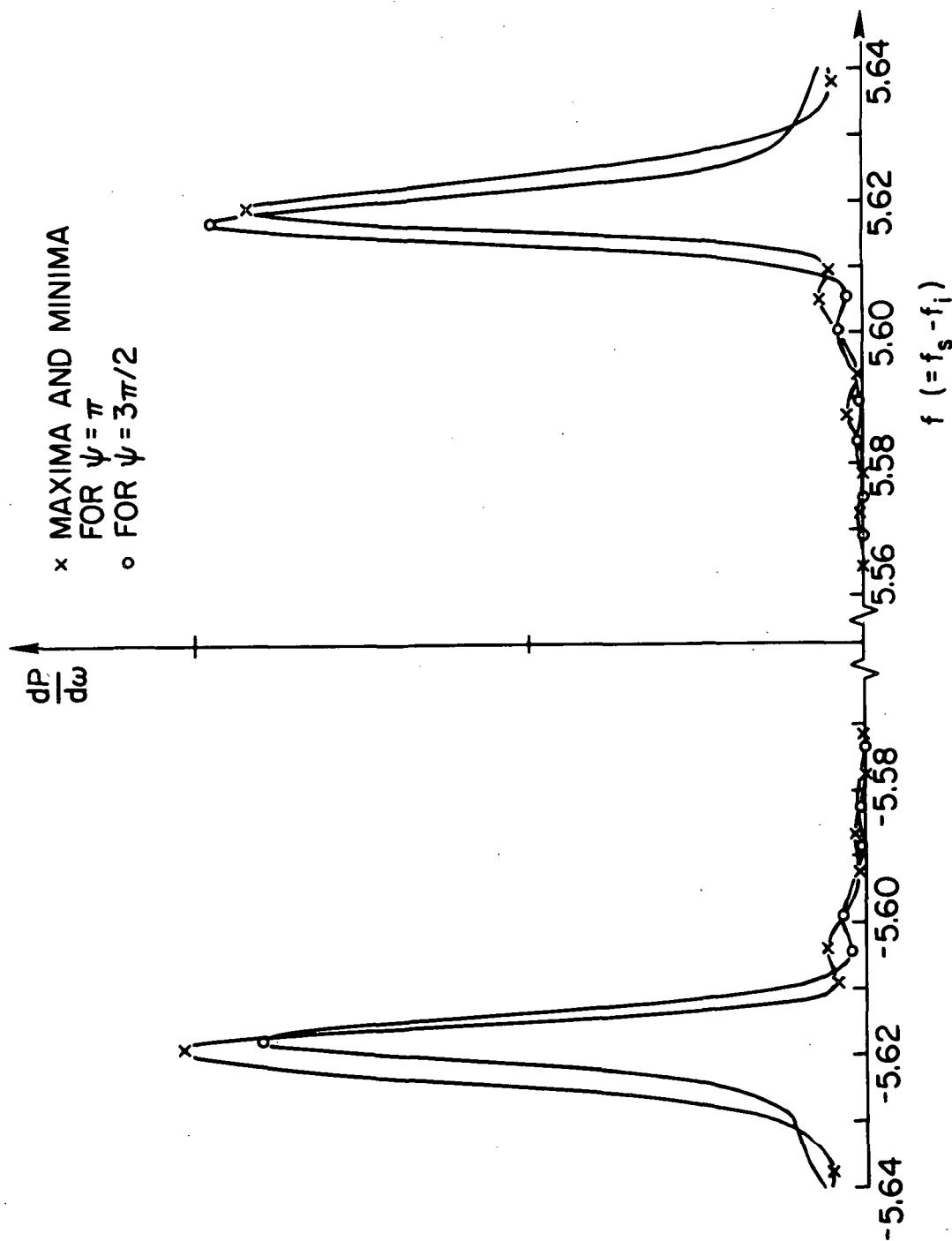


FIG. 5 (contd). (b) when  $\psi = \pi$  and  $3\pi/2$ .

## V. DISCUSSION

Our theory has been based on the subthreshold condition: in contrast to previous analyses, we have neglected nonlinear Landau damping of electron plasma waves by positive ions or effects of nonlinearly perturbed electron orbits as higher order processes. Since most of the energy deposition from the pump wave occurs at the reflection level ( $z = 1$ ), the intensity of the enhanced electron plasma wave may be well above the instability threshold. However, as shown in (26), the strongest echo received does not come from the reflection level, but from a region about 0.5 km below for typical experimental data obtained at Arecibo. In this region, the pump power from a 100 kW radar is about 0.8 times the threshold at most. In passing, we also note that the WKB approximation we have employed is valid in this region.

Inclusion of the geomagnetic field may add to the RHS of (18) an additional term  $\omega_c^2 \sin^2 \theta$ , where  $\omega_c$  is the electron cyclotron frequency due to the geomagnetic field. Though this is larger than the thermal correction term in the dispersion relation, assuming that  $\omega_c \sin \theta \ll \omega_0$ , and that the spatial dependence be much weaker than that of the wave vector  $k$ , gives (20) and (25) as good approximations. Thus, the geomagnetic effect does not invalidate the present results.

For a particular phase of the reflection coefficient, each spectrum in Figure 5 has peaks near  $f = \pm(f_0 - f_a)$ . These are due to electron plasma waves parametrically enhanced by the pump wave. The intensities of the up-shifted and down-shifted peaks differ from each other. This is due to the difference in heights where the scattered waves of  $f = \pm(f_0 - f_a)$  are generated as calculated in (26), i.e., as shown in Figure 3, there is a difference in the swelling factors at these heights. While the down-shifted peaks are smaller than the up-shifted in Figure 5 for the phases of the reflection coefficient,  $\psi = 0$  and  $3\pi/2$ , the spectra for  $\psi = \pi/2$  and  $\pi$ , where the down-shifted peaks are larger than the up-shifted, correspond to the experimental observations.

Consistent with the spectra obtained at Arecibo, there are several maxima and minima near the peak of each spectrum. These are due to the swelling factor. The frequency intervals between adjacent maxima (15  $\sim$  17 kHz) are in good agreement with the experimental results (8  $\sim$  20 kHz) (Kantor, 1972).

#### ACKNOWLEDGMENT

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